Tensile Strength and Failure Mechanics of Fibre Composites

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This paper is concerned with the prediction of the tensile strengths of fibre composites where the fibres are aligned in the direction of tensile loads, and are flawed to some extent. A theory is derived for predicting the strengths and failure mechanisms of such composites. The theory agrees reasonably well with experiments, and may be qualitatively applicable to composites containing randomly aligned fibres.

1. Introduction

The advantages of fibre reinforced materials are well known, and have been described extensively in the literature [1, 2]. In the present article, a theory is derived for predicting the tensile strengths of flawed fibre composites by combining an analysis due to Kelly [3] with a qualitative approach suggested by Parratt [4].

The following general assumptions are made: firstly, it is assumed that the fibres and matrix do not react chemically, and that good bonding occurs at their interfaces. Secondly, all the fibres are assumed to have more or less the same diameter and length. Lastly, the matrix is assumed to carry only a small amount of the direct load; it acts primarily to transfer the stress from fibre to fibre in shear. These assumptions are the usual ones made for fibre composites. They are valid for most existing composites.

2. Previous Investigations

Extensive reviews of previous investigations on the mechanics of fibre reinforcement are readily available [2, 5, 6]; therefore, only the theory relevant to the present work will be discussed.

2.1. Effect of Flaws on the Strength of Fibres

Any defects, either in the fibres or on the surface, affect the strength properties. The effects vary with the length and diameter of the fibres. The mean strength of a number of fibres in a bundle decreases with increasing length of the fibres. This is illustrated in fig. 1, [4] which shows the drop of average strength with length of the fibres.

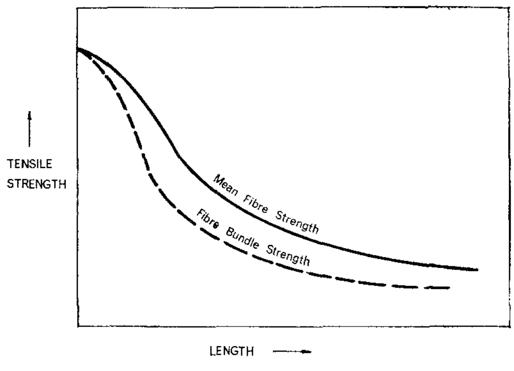

Figure 1 Effect of defects on the strengths of fibres [4].

Defects also result in a variation in strength between individual fibres, so that when a bundle of fibres is tested, some fibres will break before others. This increases the load on the remaining fibres, and the strength of the bundle is less than the mean strength of the fibres in the bundle. The dashed line in fig. 1 shows how bundle strength is related to fibre length. The bundle strength has been related to the mean strength, and to the coefficient of variation of the individual fibre strengths about the mean, by Coleman [8].

Finally, it is known that fibres become stronger with decreasing diameter. This effect has generally been attributed to the fact that, as fibres become smaller in diameter, there are fewer surface flaws on brittle fibres, and fewer dislocations in ductile fibres [7].

Many fibres in common use, such as large diameter fibres of tungsten or steel, have very few flaws. These exhibit negligible strength variation with length, or between fibre strength and bundle strength, for fibres of a given diameter. Within the present context, such fibres will be termed "flawless" and all others "flawed".

2.2. Tensile Strength of Aligned, Discontinuous Fibre Composites

A schematic representation of an aligned, discontinuous fibre composite subjected to tension is shown in fig. 2. Kelly [3] (see also [2] and [9]) has developed a theory to predict the tensile strength of such a composite where the

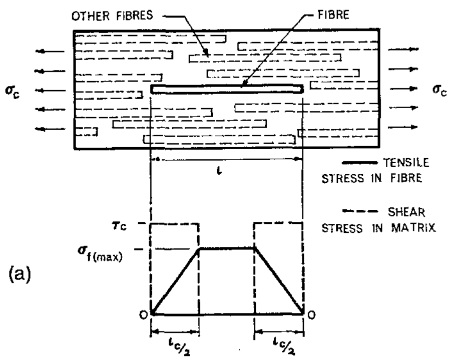

Figure 2 Schematic representation of an aligned, discontinuous fibre composite subjected to tension; (a) shows stress distribution at failure.

fibres are flawless (as defined above), and the matrix is ductile. The theory assumes that the stress distribution in the composite *at failure* has the form shown in fig. 2a. Kelly argues that high shearing stresses in the matrix near the fibre ends cause yielding of the matrix. This results in a constant shear stress τ_c in the matrix along the fibre ends, which is equal to the yield shear stress of the matrix τ_y. This constant shear stress is accompanied by a growth of tensile stress from the ends of the fibre, with a constant maximum value along the central length.

For a brittle matrix, Outwater [10] has argued that high shearing stresses in the matrix near the fibre ends cause cracking at the fibre/matrix interface. However, provided that the coefficient of shrinkage of the matrix is greater than that of the fibres, then during curing the matrix will exert a permanent compressive (shrinkage) stress on the fibres. Stress is therefore transferred over the cracked or "unbonded" areas by a constant stress, τ_c equal to a frictional stress, τ_b. Thus, an identical build-up of stress occurs for a brittle matrix as for the ductile case (fig. 2a). The implication of Outwater's argument is that Kelly's strength theory should also be applicable to fibre composites containing brittle matrices, such as fibre-glass.

Kelly, in developing his strength theory, shows that two modes of failure can occur for the composite (fig. 2a). If the length of the fibres (l) is greater than a certain critical length (l_c) then, on increasing the tensile load (σ_c), the ultimate strength of the fibres is reached, and they fracture simultaneously. The load thrown on the matrix then exceeds its ultimate strength (σ_{mu}), and results in complete failure of the composite. The breaking strength of the composite (σ_{cu}) is given by the equation

$$\sigma_{cu} = \left(1 - \frac{l_c}{2l}\right) \sigma_{fu} v_f + \sigma_{\bar{m}} v_f \qquad (1)$$

for $l \geqslant l_c$, and where $\sigma_{\bar{m}}$ is the stress borne by the matrix just before the fibres fracture. The critical length (l_c) is given by

$$l_c/d = \sigma_{fu}/2\tau_c \qquad (2)$$

for d is the fibre diameter, and, as described earlier, τ_c equals the yield shear stress (τ_y) for a ductile matrix, and a frictional stress (τ_b) for a brittle matrix.

If the fibres are shorter than the critical length, the composite will fail because the fibres pull out of the matrix at a breaking stress (σ_{cu}) given by

$$\sigma_{cu} = v_f \tau_c l/d + \sigma_{mu} v_m \qquad (3)$$

where σ_{mu} is the ultimate strength of the matrix. Equation 3 also provides a method of determining the value of the constant shear stress (τ_c) for a given brittle or ductile matrix. Thus, consider a composite in which failure occurs by pull out of the fibres ($l < l_c$). Then, knowing the failure strength (σ_{cu}), the fibre aspect ratio (l/d) and the ultimate strength of the matrix (σ_{mu}), the value of $\tau_c (= \tau_y$ or $\tau_b)$ is given by equation 3.

The above equations are only applicable provided that the volume fractions of fibres are greater than certain critical volume fractions [2]. For practical composites, these critical volume fractions are less than 10% and therefore rarely occur in practice. It is also apparent that the above equations would not be applicable for composites containing flawed fibres since, as described earlier, some fibres will break before others, due to the strength variation effect. These shorter lengths will then be stronger due to the length/strength effect, and may sustain greater loads before fracturing again. This is essentially the argument of Parratt [4], who has proposed a theory to account for the strength of flawed fibre composites; in particular, for fibre-glass. Parratt suggests that continuous fracturing of the fibres occurs until the fibres are not long enough to sustain transfer of their breaking strength, and final failure occurs by fibres pulling out of the matrix. However, Parratt does not provide more than rough quantitative expressions for his theory.

Rosen [5] has also evaluated the strength of such composites as a function of the fibre population, taking into account that near a discontinuity (fibre-end), a part of the fibre is not fully loaded. The fibres are assumed to fracture at random positions and, when enough fractures have accumulated over a given cross-section, this is a sufficient weakness for total fracture to occur. Rosen carried out an experimental study using a glass reinforced resin, but the strengths obtained were considerably less than he had predicted. This perhaps indicates that Parratt's proposal for the mode of failure is the more accurate one, at least for glass reinforced resin (fibre-glass) composites.

3. Derivation of Strength Theory

In this section, it will be shown that the analysis of Kelly [3] can be combined with the qualitative approach of Parratt [4] (see section 2.2) to provide a theory for predicting the tensile strength and failure mechanism of a fibre composite where the fibres are aligned in the direction of tensile loads, and are flawed to some extent. As noted in section 2.1, the strength of a bundle of flawed fibres decreases with increasing length of the fibres as shown by the dashed line in fig. 1. This curve can be expressed as

$$\sigma_{fu} = f(l) \qquad (4)$$

where σ_{fu} is the breaking strength of a bundle of fibres of a given diameter, and having a current average length l. This relation can be determined for a given fibre composite by obtaining experimental length/strength data for the single fibres and relating the results to bundle strength, using the theoretical results of Coleman [8]. Given equation 4, equations 5, 6, and 7 follow directly from equations 1, 2, and 3 respectively.

$$l_c/d = f(l)/2\tau_c \qquad (5)$$

$$\sigma_{cu} = \left(1 - \frac{l_c}{2l}\right) f(l) v_f + \sigma_{\bar{m}} v_m \qquad (6)$$

for $l \geqslant l_c$.

$$\sigma_{cu} = v_f \tau_c l/d + \sigma_{mu} v_m \qquad (7)$$

for $l \leqslant l_c$, and where σ_{cu} is the strength of the composite, d is the fibre diameter, l_c is the critical length of the fibres. Equation 6 will generally define one of the three types of curves (a-b) shown in fig. 3 depending on the nature of equation 4. Equation 7 will define the linear pull-out curve (c-d) also shown in fig. 3. Assuming that the fibres have the same initial length, l_i, where $l_i \gg l_c$, the curves (a-b) can be used to predict the failure mechanism and strength of a given fibre composite as follows. Note that the numerical values shown in fig. 3 have no significance at present, and are used later when the theory is compared with experiment.

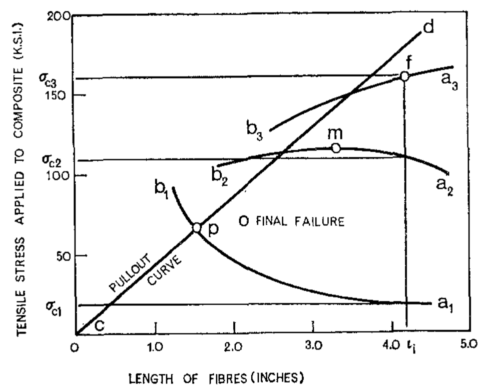

Figure 3 Modes of failure for flawed fibre composites. *Note* 1 in. = 2.5 cm; 1 ksi = 1000lb/in.^{2} = 70.3 kg/cm^2.

Case 1 (a_1-b_1) This shape of curve occurs for badly flawed fibres; that is, where there is a severe length/strength effect. When the composite reaches the stress σ_{c1} (fig. 3) the fibres will fracture. However, since the curve slopes upwards for shorter lengths of the fibres the ultimate load of the composite is far from reached. The broken fibres, which are shorter and therefore "stronger", will not fracture again until the load is increased. Increasing the load is equivalent to moving up the curve until the point p is reached, where the average length of the fibres equals the critical length (l_c) given by equation 5; and at this point the composite fails because the fibres pull out of the matrix. The strength of the composite (σ_{cu}) is therefore given from equation 7 as

$$\sigma_{cu} = v_f \tau_c l_c / d + \sigma_{mu} v_m \quad (8)$$

for $l_i \geqslant l_c$, case 1, and the critical or failing length (l_c) is given by equation 5. Equation 8 can be expressed more conveniently using equations 4 and 5 as follows

$$\sigma_{cu} = \sigma_{fu} v_f/2 + \sigma_{mu} v_m \quad (9)$$

for $l_i \geqslant l_c$, case 1.

Case 2 (a_2-b_2) This shape of curve could arise for lightly flawed fibres, and is characterised by the maximum point m before the curve intersects the linear pull-out curve (c-d). As for case 1, fracturing of the fibres will begin when the composite reaches the stress σ_{c2}, and will continue until complete failure occurs at the maximum point (m). The strength of this type of composite is given by equation 6 where the average length of the fibres at final failure is given by $d\sigma_{cu}/dl = 0$, for $l_i \gg l_c$, case 2.

Case 3 (a_3-b_3) This shape of curve would arise for flawless or lightly flawed fibres. When the composite reaches the stress σ_{c3}, the fibres will fracture, and failure will be final at this point (f), since the curve slopes downwards for shorter lengths of the fibres. The strength of this type of composite is given by equation 6 where the length of fibres at final failure is simply their initial length (l_i).

When $l_i \leqslant l_c$, failure will be by immediate pull-out of the fibres, and the strength of the composite is given by equation 7 where $l = l_i$.

The continuous fracturing of the fibres which occurs with increasing load for cases 1 and 2 when $l_i \gg l_c$ explains why snapping noises are heard in flawed fibre composites such as fibreglass during the first loading cycle. An important conclusion is that if a composite is characteristically case 1 (badly flawed fibres), then the fibres can never contribute more than 50% of their maximum inherent strength to the strength of the composite. That is, since final failure is by pulling out of the fibres at the critical length l_c, then from equation 7, the contribution of the fibres to the strength of the composite is the quantity $v_f \tau_c l_c / d$, and from equations 4 and 5,

$$v_f \tau_{mc} l_c / d = f(l_c) v_f / 2 = \sigma_{fu} v_f / 2$$

4. Comparison with Experiment

A search of the literature by the authors revealed that realistic comparisons with the theory could only be made with the results of Parratt (4) on resins reinforced with aligned glass rovings. Other experimental data is characterised by either excessive scatter, or insufficient data. Parratt's specimens were tested in simple tension in the direction of alignment of the fibres. The specimens were fabricated using different diameters of fibres. Tensile strengths on the individual glass fibres showed that the strengths of the fibres rose from 80 ksi (1 ksi = 1000 lb/in.^{2} = 70.3 kg/cm^2) for 2 in. (1 in. = 2.5 cm) lengths up to 300 ksi for lengths below 0.25 in. and did not vary appreciably with the diameter of the fibres. Since this is a severe length/strength effect, the fibres are flawed (as defined in section 2.1). Parratt did not provide data for the scatter of strengths of the fibres of a given diameter, so we assumed that a bundle of these fibres behaves approximately the same as the single fibres. If it is further assumed that the length/strength effect is linear, the relation expressed by equation 4 is given as

$$\sigma_{fu} = f(l) = 300 - 110l \quad (10)$$

When equation 10 is substituted in equations 6 and 7, and plotted for the various diameters of fibres used in the specimens, it is found that all of the specimens are characteristically case 1 (badly flawed fibres, section 3). In fact, the curves (a_1-b_1) and (c-d) in fig. 3 are really plots of equations 6 and 7 respectively where σ_{fu} is given by equation 10, and where the diameter of the fibres (d) equals 0.01 in.

Having established that the mode of failure of the composites is case 1, it is now possible to predict the strength of a given composite directly from equation 9 as follows. Note that the contribution of the matrix to the strengths of the composites ($\sigma_{mu} v_m$) is small, and will therefore be neglected. The value of the constant frictional stress (τ_b) (resins are brittle, and therefore $\tau_c = \tau_b$, section 2.2) can be found from fig. 4 which is a reproduction of Parratt's fig. 5. From section 3, the curve of fig. 4 should be a linear pull-out curve defined by equation 7 where

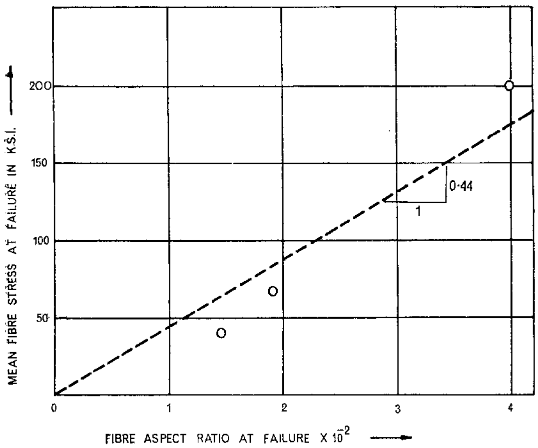

Figure 4 Derivation of the frictional stress (τ_b) from the experimental results of Parratt [4]. *Note* 1 ks = 1000 lb/in.^{2} =70.3 kg/cm^2.

$v_f = 1$ (Parratt has corrected to 100% volume fraction of fibres for his curves). Therefore, a straight line (dashed) has been drawn through the mean of the experimental points, and has a measured slope of 0.44 ksi. From equation 7, this slope equals τ_b; that is

$$\tau_b = 0.44 = \sigma_{cu}\, d/l_c \qquad (11)$$

for $v_f = 1$. Then, from equations 5 and 10

$$l_c/d = \frac{300 - 110 l_c}{2 \times 0.44} \qquad (12)$$

and from equation 9 the strength of a given composite containing fibres of diameter d is given by

$$\sigma_{cu} = \sigma_{fu}/2 \qquad (13)$$

for $v_f = 1$, and σ_{fu} is evaluated from equation 10 with $l = l_c$. Equations 10, 12, and 13 then completely define the strength of the composites. For example, for the specimen containing fibres of diameter $d = 0.01$ in., the final pull-out length is given by equation 11 as

$$l_c/0.01 = \frac{300 - 110 l_c}{2 \times 0.44}$$

i.e. $l_c = 1.52$ in. and the composite will fail by pull-out of the fibres, when the fibres have fractured to the average length of 1.52 in. The strength of the composite is given from equations 10 and 12 as

$$\sigma_{cu} = \sigma_{fu}/2 = (300 - 110 l_c)/2 = (300 - 110 \times 1.52)/2 = 66.5.$$

Similarly, the theoretical values of σ_{cu} can be found for the whole range of diameters of fibres, and provide the theoretical curve (dashed) shown in fig. 5 which is a reproduction of Parratt's fig. 4. Note that good agreement is obtained with Parratt's results for the complete range of diameters; and as predicted by the theory of section 3, the maximum strength of the composites (150 ksi) is 50% of the maximum inherent strength of the fibres (300 ksi) (a direct comparison can be made since Parratt has corrected to 100% volume fraction of fibres as mentioned above).

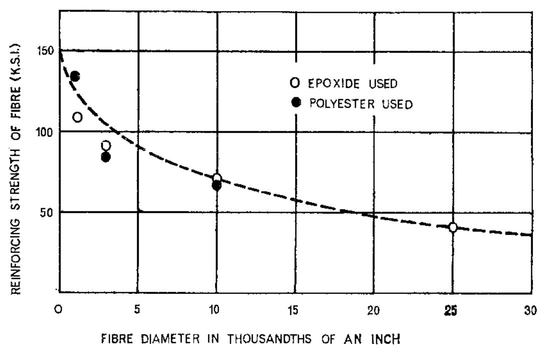

Figure 5 Comparison of theory (dashed line) with the experimental results of Parratt [4]. *Note* 1 in. = 2.5 cm; 1 ksi = 1000 lb/in.^{2} = 70.3 kg/cm^2.

5. Conclusions

It appears that the theory derived in section 3 provides good approximations for the tensile strengths of aligned, flawed fibre composites. It was seen that several modes of failure can occur for such composites depending on the initial length of the fibres, and on the degree to which the fibres are flawed. Most fibres are handled during manufacture and fabrication and become badly flawed; therefore, this case is of special interest. For this case, the composite fails by a successive fracturing of the fibres until final failure by pulling out of the fibres. Furthermore, badly flawed fibres can never contribute more than 50% of their maximum inherent strength to the strength of the composite.

The composite strength was shown to be primarily dependent on three factors: these are the degree to which the fibres are flawed, the aspect ratio of the fibres, and the transfer stress developed by the matrix. From this point of view, ductile or semi-ductile metals are probably more suitable for use as matrices for the newer whiskers which have comparatively low aspect ratios. That is, from equations 6, 7, and 8, the high strengths of the whiskers can be developed by a matrix with a high transfer stress (τ_c) since this compensates for a low aspect ratio (l/d). Ductile materials develop a high transfer stress equal to their yield shear stress whereas brittle materials such as resins can develop only comparatively low frictional stresses.

It may be that the theory developed can also be applied to predict the tensile strengths of flawed fibre composites, where the fibres are orientated in random directions by using the equations developed by Cox [11]. However, this requires further investigation. Certainly, the theory should be applicable for providing qualitative trends for the tensile strengths of such composites.

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